South Sefton Campus
Maths Department

Preparation for A-level Maths

Work through all sections of this booklet over the summer holiday, checking your answers as you go; then complete the homework at the back and hand this is in at your first maths lesson at the college in September.

Contents

Section 1 Equivalent Fractions

The fractions $\frac{2}{3}$, $\frac{4}{6}$ and $\frac{10}{15}$ are **equivalent fractions.**

To find an equivalent fraction multiply the numerator (top) and the denominator (bottom) by the same number:

$$\overset{\times 2}{\frac{2}{3} = \frac{4}{6}}_{\times 2} \qquad \overset{\times 3}{\frac{3}{5} = \frac{9}{15}}_{\times 3} \qquad \overset{\times 4}{\frac{3}{7} = \frac{12}{28}}_{\times 4}$$

Some fractions can be simplified by **cancelling**:

$$\overset{\div 5}{\frac{20}{25} = \frac{4}{5}}_{\div 5} \qquad \overset{\div 8}{\frac{16}{24} = \frac{2}{3}}_{\div 8} \qquad \overset{\div 7}{\frac{21}{28} = \frac{3}{4}}_{\div 7}$$

Exercise 1

1. Find the missing numbers:

(a) $\frac{2}{5} = \frac{}{10}$ (b) $\frac{3}{4} = \frac{}{8}$ (c) $\frac{2}{3} = \frac{}{9}$

(d) $\frac{2}{7} = \frac{6}{}$ (e) $\frac{3}{5} = \frac{12}{}$ (f) $\frac{7}{10} = \frac{21}{}$

2. Simplify these fractions:

(a) $\frac{9}{12}$ (b) $\frac{8}{20}$ (c) $\frac{32}{36}$ (d) $\frac{27}{45}$

(e) $\frac{56}{64}$

Section 2 Adding and Subtracting Algebraic Fractions

Example 2a Simplify $\frac{3x}{4}+\frac{2x}{5}$

× 5 × 4

$$\frac{3x}{4}+\frac{2x}{5} = \frac{5\times 3x}{20}+\frac{4\times 2x}{20}$$

× 5 × 4

Common denominator is least common multiple of 4 and 5 which equals 20.

Whatever we do to the bottom of a fraction we must also do to the top of that fraction.

$$= \frac{15x+8x}{20}$$

$$= \frac{23x}{20}$$

Example 2b Write $\frac{2}{x+3}-\frac{5}{x-1}$ as a single fraction

$$\frac{2}{x+3}-\frac{5}{x-1} = \frac{2(x-1)-5(x+3)}{(x+3)(x-1)}$$

$$= \frac{2x-2-5x-15}{(x+3)(x-1)}$$

$$= \frac{-3x-17}{(x+3)(x-1)}$$

Least common multiple of (x+3) and (x-1) is (x+3)(x-1).

Multiply top and bottom of first fraction by (x-1).
Multiply top and bottom of second fraction by (x+3).

Exercise 2

1. Simplify:

(a) $\frac{3x}{4}-\frac{2x}{3}$ (b) $\frac{x}{2}+\frac{x+1}{3}$ (c) $\frac{x-1}{3}+\frac{x+2}{4}$

2. Write as a single fraction:

(a) $\frac{1}{x}+\frac{2}{x+1}$ (b) $\frac{5}{x-2}+\frac{3}{x+3}$ (c) $\frac{3}{x-2}-\frac{4}{x+1}$

Section 3 Multiplying out brackets

Example 3a Expand $b(2a + 3b - c)$

$b(2a + 3b - c) = 2ab + 3b^2 - bc$

Example 3b Expand and simplify:

(a) $2(x - 3) + x(x + 4)$

$2(x - 3) + x(x + 4) = 2x - 6 + x^2 + 4x = x^2 + 6x - 6$

(b) $6y - 2(y - 3)$

$6y - 2(y - 3) = 6y - 2y + 6 = 4y + 6$

Be careful when multiplying a bracket by a negative number; all terms in the bracket are multiplied by -2

Exercise 3 Expand, and simplify where possible:

1 $2(2x + 1)$

2 $x(2y + z)$

3 $a(a - 3)$

4 $-2(2x + 3)$

5 $-3(a - 2)$

6 $p(q - p)$

7 $3(x + 2) + 4(x + 1)$

8 $4(2x + 3) - 3(3x - 2)$

Section 4 Factorising

When factorising we look for **common factors**.

Example 4

Factorise (a) $x^2 + 7x$ (b) $3y^2 - 12y$ (c) $6a^2b - 10ab^2$

(a) x is common to x^2 and $7x$, so $x^2 + 7x = x(x + 7)$

(b) $3y$ is common to $3y^2$ and $12y$, so $3y^2 - 12y = 3y(y - 4)$

(c) $2ab$ is common to $6a^2b$ and $10ab^2$, so $6a^2b - 10ab^2 = 2ab(3a - 5b)$

Exercise 4 Factorise as far as possible:

1 $x^2 + 5x$

2 $7x - x^2$

3 $6c^2 - 21c$

4 $ax + bx + 2cx$

5 $x^2y + xy^2$

6 $6a^2 + 4ab + 2ac$

7 $5x - 10x^3$

8 $3\pi r^2 + \pi rh$

Section 5 Solving linear equations

Example 5 Solve the following equations:

(a) $5x - 9 = 12 - 4x$

(b) $3(x - 2) = 2(x + 6)$

(c) $\frac{4x+1}{3} = x$

(a) $5x - 9 = 12 - 4x$

$5x + 4x = 12 + 9$

Get all the x terms on one side and all the other terms on the other side, by moving $-4x$ on the right hand side to become $+4x$ on the left hand side, and moving -9 on the left hand side to become $+9$ on the right hand side.

$9x = 21$ (collect together like terms)

$x = \frac{21}{9}$ (divide by 9 to give value of x)

(b) $3(x - 2) = 2(x + 6)$

$3x - 6 = 2x + 12$ (multiply out brackets)

$3x - 2x = 12 + 6$ (x terms on one side, numbers on other side)

$x = 18$ (collect together like terms)

(c) $\frac{4x+1}{3} = x$

$4x + 1 = 3x$ (multiply both sides by 3)

$4x - 3x = -1$ (subtract $3x$ from both sides, subtract 1 from both sides)

$x = -1$ (collect together like terms)

Exercise 5 Solve the following equations:

1 $4 - 3x = 2$

2 $5x - 4 = 3 - x$

3 $3(x + 2) = x + 4$

4 $5(x - 2) + 6 = 3(x - 4) + 10$

5 $\frac{7x-3}{2} = x$

6 $\frac{x}{3} + 7 = 5$

Section 6 Solving linear inequalities

Inequalities can use any of the following symbols:

$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to

Inequalities can be manipulated in much the same ways as equations *(taking from or adding to both sides, multiplying both sides by a number)*, except that:

Multiplying or dividing an inequality by a negative number reverses the inequality *(try to avoid doing this!)*

Example 1 Solve the inequality $2x + 1 > 6$

$2x > 6 - 1$ (subtract 1 from both sides)

$2x > 5$

$x > \frac{5}{2}$ (divide both sides by 2)

Example 2 Solve the inequality $3 - 5x \geq 7 - 3x$

to avoid needing to divide by a negative number later on, move the 5x to the right hand side so that we have +2x

$3 - 7 \geq -3x + 5x$

$-4 \geq 2x$

$\frac{-4}{2} \geq x$ (divide both sides by 2)

$x \leq -2$ (rewrite so that the x in on the left hand side, but remember that the inequality will now be the other way round)

Exercise 6 Solve the following inequalities:

1 $2x - 3 < 9$

2 $4x - 2 \leq 2x + 6$

3 $2(x + 1) > x - 7$

4 $7 < 15 - x$

5 $3(x - 1) \geq 2(1 - x)$

6 $\frac{2x}{5} > 3$

Section 7 Expanding double brackets

There are several different methods for multiplying two brackets. Here are some of them.

Example 7a Table

$(x+3)(x-2)$

	x	$+3$
x	x^2	$+3x$
-2	$-2x$	-6

So, $(x+3)(x-2) = x^2 + 3x - 2x - 6 = x^2 + x - 6$

Example 7b 'Smiley Face'/FOIL

$(2x+1)(x-3)$ $\qquad (2x+1)(x-3) = 2x^2 - 6x + 1x - 3 = 2x^2 - 5x - 3$

Either, use the **smiley face** picture and multiply the terms at the ends of each line together,

or, use **FOIL**, standing for first (*first term in each bracket*), outside (the two terms on the outside i.e. 2x and -3), inside (the two terms on the inside i.e. 1 and x) and last (the last term in each bracket).

Exercise 7 Expand the brackets and simplify:

1 $(x+4)(x+3)$

2 $(3x+1)(x+2)$

3 $(2x-3)(2x-5)$

4 $(4x-1)(x+5)$

5 $(x+2)^2$ *(Note: for squared brackets it is advisable to write out as two separate brackets, e.g. $(x+2)^2 = (x+2)(x+2)$, and then multiply out.)*

6 $(x-7)^2$

Section 8 Factorising quadratics

If we are attempting to factorise a quadratic function of the form $x^2 + bx + c$, we need to find two numbers which add to give b, the number of x, and multiply to give c, the number on its own.

Example 8a Factorise $x^2 + 3x + 2$.

The two numbers must add up to 3 and multiply together to give 2. They are, therefore, 1 and 2, so that

$$x^2 + 3x + 2 = (x + 1)(x + 2)$$

You must also be aware of the following special cases.

Common Factors

If there is no constant term, then x is a **common factor**:

Example 8b Factorise $x^2 - 6x$

x is a common factor, so that $x^2 - 6x = x(x - 6)$

The Difference of Two Squares

Any expression of the form $x^2 - k^2$ factorises to $(x + k)(x - k)$

Example 8c Factorise $x^2 - 9$

$$x^2 - 9 = (x + 3)(x - 3)$$

Quadratics with more than a single x^2 (e.g.$3x^2 + 4x + 1$) are more difficult and can require a lot of trial and error.

Further examples:

Example 8d Factorise $x^2 - x - 12$.

The two numbers which add up to -1 and multiply to give -12 are -4 and $+3$, so that

$$x^2 - x - 12 = (x - 4)(x + 3)$$

Example 8e Factorise $x^2 - 1$.

This is the difference of two squares. The squares are x^2 and 1^2, so that

$$x^2 - 1 = (x + 1)(x - 1)$$

Example 8f Factorise $2x^2 - x - 3$

Although this is of the most difficult type the numbers are quite helpful. To get the $2x^2$ at the start we can only have $2x \times x$ and -3 must be either 3×-1 or -3×1 so we try these out in the brackets

$$(2x \quad\quad)(x \quad\quad)$$

None of the combinations $(2x + 3)(x - 1)$, $(2x - 1)(x + 3)$ or $(2x + 1)(x - 3)$ work *(you can check this by multiplying out the brackets)* so we are only left with $(2x - 3)(x + 1)$ which works.

Exercise 8 Factorise the following:

1	$x^2 + 5x + 6$	2	$x^2 + x - 2$	3	$x^2 - 6x + 8$
4	$x^2 + 18x$	5	$x^2 - 16$	6	$2x^2 + 5x + 2$
7	$x^2 - x$	8	$5x^2 - x - 4$	9	$2x^2 + 8x$

Section 9 Solving quadratics by factorising

We can use *factorisation*, *the quadratic formula* and *completing the square* to solve quadratic equations. We will now look at just the first of these methods.

Example 9 Solve the equation $2x^2 + x - 6 = 0$.

The left hand side can be factorised to $(2x - 3)(x + 2)$ so that the equation becomes

$$(2x - 3)(x + 2) = 0$$

$\Rightarrow$ Either $2x - 3 = 0$ or $x + 2 = 0$

$\Rightarrow$ $2x = 3$ $x = -2$

$\Rightarrow$ $x = \frac{3}{2}$

Exercise 9 Solve the following:

1 $x^2 - 6x + 5 = 0$

2 $x^2 - 5x - 14 = 0$

3 $x^2 + x - 12 = 0$

4 $6x^2 + x - 2 = 0$

5 $x^2 - 7x = 0$

6 $x^2 - 100 = 0$

7 $2x^2 - 3x - 2 = 0$

8 $x^2 + 2x + 1 = 0$

Section 10 Formulae

Rearranging formulae

Example 10a If $c = kf - x$, make f the subject.

$c + x = kf$ *(add x to both sides)*

$\frac{c+x}{k} = f$ *(divided both sides by k)*

i.e. $f = \frac{c+x}{k}$ *(write with f on the left hand side)*

Example 10b If $b = \frac{c-y}{d+y}$, make y the subject.

$b(d + y) = c - y$ (multiply both sides by $d + y$ to get rid of fraction)

$bd + by = c - y$ (expand brackets)

$by + y = c - bd$ (get all y-terms on one side)

$y(b + 1) = c - bd$ (factorise left hand side)

$y = \frac{c-bd}{b+1}$ (divide by $b + 1$ to leave y on its own)

Replacing letters with numbers is called **substitution**. Remember to use BIDMAS or BoDMAS.

Example 10c Find the value of $(2p + q)^3$ when $p = 3$ and $q = 1$.

$$(2p + q)^3 = (2 \times 3 + 1)^3$$

$$= (6 + 1)^3$$

$$= 7^3$$

$$= 343$$

Exercise 10

1. The tension, T, in a particular spring is given in terms of its extension, x, and original length, l, by the equation $T = \frac{5x}{l}$.

   (a) Find T when $x = 20$ and $l = 25$.

   (b) Rearrange the above formula to give x in terms of T and l.

   (c) Using your new formula, find x when $T = 7$ and $l = 10$.

2. If $a = m(c + 2)$

   (a) Find a when $c = 5$ and $m = 3$.

   (b) Rearrange the above formula to make c the subject.

   (c) Using your new formula, find c when $a = 12$ and $m = 12$.

3. The energy, k, of a moving object is related to its mass, m, and speed, v, by the equation $k = \frac{1}{2}mv^2$.

   (a) Find k when $m = 2$ and $v = 4$.

   (a) Rearrange the above formula to make v the subject.

   (c) Using your new formula, find v when $k = 90$ and $m = 5$.

Section 11 Rules of indices

To multiply powers of the same number, add the indices

e.g. $7^3 \times 7^5 = 7^{3+5} = 7^8$

To divide powers of the same number, subtract the indices

e.g. $10^6 \div 10^2 = 10^{6-2} = 10^4$

To raise the power of a number to another power, multiply the indices

e.g. $(3^5)^2 = 3^{5\times 2} = 3^{10}$

Example 11 Simplify:

(a) $3y^2 \times 4y^3 = 12y^5$

(b) $(3x^2)^3 = 3^3 \times (x^2)^3 = 27x^6$

(c) $8x^3y^4 \div 2x^2y = 4x^{3-2}y^{4-1} = 4xy^3$

Exercise 11 Simplify:

1 $x^7 \times x^{13}$

2 $x^3 \div x^7$

3 $(x^4)^3$

4 $2x^5y^2 \times 5x^3y^7$

5 $(4x^3y^2)^2$

6 $3x^7y^2 \div x^5y^4$

7 $\left(2x^{\frac{1}{3}}\right)^3$

8 $5x^3y \div 5x^2y$

Section 12 Simultaneous equations

Substitution Method

Example 12a Solve the equations $3x - 2y = 0$ and $2x + y = 7$.

$3x - 2y = 0$ [1]
$2x + y = 7$ [2]

From equation 2: $y = 7 - 2x$

Substitute for y in equation 1: $3x - 2(7 - 2x) = 0$

$3x - 14 + 4x = 0$

$7x = 14$

$x = 2$

Sub. for $x = 2$ in rearranged equation 2: $y = 7 - 2 \times 2 = 7 - 4 = 3$

Elimination Method

Example 12b Solve the equations $2x + 3y = 5$ and $5x - 2y = -16$.

$2x + 3y = 5$ [1]
$5x - 2y = -16$ [2]

$[1] \times 5$ $10x + 15y = 25$ [3]
$[2] \times 2$ $10x - 4y = -32$ [4]

$[3] - [4]$ $0x + 19y = 57$

$$y = \frac{57}{19} = 3$$

Substitute for x in equation $[1]$: $2x + 3 \times 3 = 5$

$2x + 9 = 5$

$2x = 5 - 9$

$2x = -4$

$$x = \frac{-4}{2} = -2$$

Exercise 12

1. Solve using the substitution method:

(a) $2x + y = 5$
$x + 3y = 5$

(b) $x + 2y = 8$
$2x + 3y = 14$

(c) $3x + y = 10$
$x - y = 2$

(d) $2x + y = -3$
$x - y = 2$

2 Solve using the elimination method:

(a) $2x + 5y = 24$
$4x + 3y = 20$

(b) $3x + 2y = 11$
$2x - y = -3$

(c) $2x + 7y = 17$
$5x + 3y = -1$

(d) $5x - 7y = 27$
$3x - 4y = 16$

Answers to exercises

Exercise 1

1.	(a)	4	(b)	6	(c)	6	(d)	21	(e)	20	(f)	30
2.	(a)	$\frac{3}{4}$	(b)	$\frac{2}{5}$	(c)	$\frac{8}{9}$	(d)	$\frac{3}{5}$	(e)	$\frac{7}{8}$		

Exercise 2

1.	(a)	$\frac{x}{12}$	(b)	$\frac{5x+2}{6}$	(c)	$\frac{7x+2}{12}$
2.	(a)	$\frac{3x+1}{x(x+1)}$	(b)	$\frac{8x+9}{(x-2)(x+3)}$	(c)	$\frac{11-x}{(x-2)(x+1)}$

Exercise 3

1	$4x+2$	2	$2xy+xz$	3	a^2-3a	4	$-4x-6$
5	$-3a+6$	6	$pq-p^2$	7	$7x+10$	8	$-x+18$

Exercise 4

1	$x(x+5)$	2	$x(7-x)$	3	$3c(2c-7)$	4	$x(a+b+2c)$
5	$xy(x+y)$	6	$2a(3a+2b+c)$	7	$5x(1-2x^2)$		
8	$\pi r(3r+h)$						

Exercise 5

1	$x=\frac{2}{3}$	2	$x=\frac{7}{6}$	3	$x=-1$	4	$x=1$
5	$x=\frac{3}{5}$	6	$x=-6$				

Exercise 6

1	$x<6$	2	$x\le 4$	3	$x>-9$	4	$x<8$
5	$x\ge 1$	6	$x>\frac{15}{2}$				

Exercise 7

1	$x^2+7x+12$	2	$3x^2+7x+2$	3	$4x^2-16x+15$
4	$4x^2+19x-5$	5	x^2+4x+4	6	$x^2-14x+49$

Exercise 8

1	$(x+2)(x+3)$	2	$(x+2)(x-1)$	3	$(x-4)(x-2)$
4	$x(x+18)$	5	$(x+4)(x-4)$	6	$(2x+1)(x+2)$
7	$x(x-1)$	8	$(5x+4)(x-1)$	9	$2x(x+4)$

Exercise 9

1	$x = 1 \; or \; x = 5$	2	$x = 7 \; or \; x = -2$
3	$x = -4 \; or \; x = 3$	4	$x = -\frac{2}{3} \; or \; x = \frac{1}{2}$
5	$x = 0 \; or \; x = 7$	6	$x = 10 \; or \; x = -10$
7	$x = -\frac{1}{2} \; or \; x = 2$	8	$x = -1$

Exercise 10

1	(a)	$T = 4$	(b)	$x = \frac{Tl}{5}$	(c)	$x = 14$
2	(a)	$a = 21$	(b)	$c = \frac{a}{m} - 2 \; or \; \frac{a-2m}{m}$	(c)	$c = -1$
3	(a)	$k = 16$	(b)	$v = \sqrt{\frac{2k}{m}}$	(c)	$v = 6$

Exercise 11

1	x^{20}	2	x^{-4}	3	x^{12}	4	$10x^8y^9$
5	$16x^6y^4$	6	$3x^2y^{-2}$	7	$8x$	8	x

Exercise 12

1	(a)	$x = 2, \; y = 1$	(b)	$x = 4, \; y = 2$
	(c)	$x = 3, \; y = 1$	(d)	$x = -\frac{1}{3}, \; y = -\frac{7}{3}$
2	(a)	$x = 2, \; y = 4$	(b)	$x = \frac{5}{7}, \; y = \frac{31}{7}$
	(c)	$x = -2, \; y = 3$	(d)	$x = 4, \; y = -1$

A-level Maths Summer Holiday Homework

Complete on A4 lined paper.

Please bring your solutions to this homework, including all working out, to your first A-level Maths lesson in September.

1 Factorise fully $x^3 - 4x^2$. (2 marks)

2 (a) Solve $6x + 9 = 2x + 7$. (3 marks)

(b) Expand and simplify $3(7a - 5b) + 2(4a - 3b)$. (3 marks)

(c) Given that $a = 8, b = -3, c = 1\ and\ d = -5$,

work out the value of $\frac{a+b}{cd}$. (3 marks)

3 (a) Simplify $8^4 \times 8^5$.

Leave your answer as a power of 8. (1 mark)

(b) Simplify $w^6 \div w^2$. (1 mark)

(c) Chris simplifies $3x \times 4x^5$.

His answer is $7x^5$.

Explain the mistakes he has made. (2 marks)

(d) Simplify fully $15y^6z^3 \div 5y^2z$. (2 marks)

4 Solve $2x^2 + 3x - 90 = 0$. (3 marks)

5 Simplify fully $\frac{4x^2-25}{6x^2-15x}$ (3 marks)

6 (a) Complete the following.

$$\frac{3}{4} = \frac{}{28}$$

$$\frac{1}{7} = \frac{}{28}$$ (2 marks)

(b) Simplify $\frac{3x}{4} + \frac{x}{7} - \frac{x}{2}$ (3 marks)

7 (a) Solve the equation $\frac{y}{4} + 1 = 3 - y$ (3 marks)

(b) List all the integer solutions of the inequality $-6 \leq 3n < 5$ (3 marks)

(c) Solve the simultaneous equations

$$4x + y = 5$$
$$3x - 2y = 12$$

Do **not** use trial and improvement.
You **must** show your working. (3 marks)

8 (a) Multiply out $x(3 - x)$ (1 mark)

(b) Expand and simplify $(2q - 7)(3q - 4)$ (3 marks)

9 Show that $(y + 2)^2 - (y - 2)^2 \equiv 8y$ (3 marks)

10 Make v the subject of the formula $f = \frac{uv}{u+v}$ (4 marks)